



# Advanced Business Mathematics

## **SAMPLE TIME CONSTRAINED ASSESSMENT MARKING SCHEME**

This marking scheme has been prepared as a **guide only** to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) **must** be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

**Throughout the marking, please credit any valid alternative point.**

**Where markers award half marks in any part of a question, they should ensure that the total mark recorded for the question is rounded up to a whole mark.**

<b>Section A</b>
<b>You must answer this question</b>

**Marks**

**Question 1**

- a) The data below shows the annual *per capita* (per person) consumption of three items over a two-year period.

	2019		2020	
	Price \$	Quantity (kg)	Price (\$)	Quantity (kg)
Item 1	8.26	15 520	9.10	16 100
Item 2	3.85	9520	3.95	9420
Item 3	12.58	7850	13.15	8150

Using **2019** as the base year calculate:

- i) the *Laspeyres price index* for 2020. Give your answer to TWO (2) decimal places. **3**

**Mark scheme**

$$\begin{aligned}
 LPI &= \frac{\sum q_1 p_2}{\sum q_1 p_1} \times 100 \\
 &= \frac{(15520 \times 9.10) + (9520 \times 3.95) + (7850 \times 13.15)}{(15520 \times 8.26) + (9520 \times 3.85) + (7850 \times 12.58)} \times 100 \\
 &= \frac{282063.5}{263600.2} \times 100 \\
 &= 107.00 \text{ (to 2 d.p)}
 \end{aligned}$$

**(1 mark for partially correct working, 2 marks for wholly correct working. 1 mark for correct answer.)**

- ii) the *Paasche price index* for 2019. Give your answer to TWO (2) decimal places. **3**

**Mark scheme**

$$\begin{aligned}
 PPI &= \frac{\sum q_2 p_2}{\sum q_2 p_1} \times 100 \\
 &= \frac{(16100 \times 9.10) + (9420 \times 3.95) + (8150 \times 13.15)}{(16100 \times 8.26) + (9420 \times 3.85) + (8150 \times 12.58)} \times 100 \\
 &= \frac{290891.5}{271780} \times 100 \\
 &= 107.03 \text{ (to 2 d.p)}
 \end{aligned}$$

**(1 mark for partially correct working, 2 marks for wholly correct working. 1 mark for correct answer.)**

b) For the equation:

$$y = 5x^2 \ln(2x)$$

Differentiate  $y$  with respect to  $x$ .

**Mark scheme**

**Using the product rule:**  $\frac{d}{dx} (f(x)g(x)) = f(x) \frac{dg}{dx} + g(x) \frac{df}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= 5x^2 \times \frac{1}{x} + 10x \ln(2x) \\ &= 5x + 10x \ln(2x) \end{aligned}$$

**(Award 1 mark for using product rule and 1 mark for each correct term.)**

c) Calculate the *Pearson Correlation Coefficient* for the set of sample observations given in the table below.

7

$x$	25	31	37	39	42	46
$y$	89	71	79	68	67	52

**Mark scheme**

**Using:**

$$R = r = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{(n\sum x_i^2 - (\sum x_i)^2)(n\sum y_i^2 - (\sum y_i)^2)}}$$

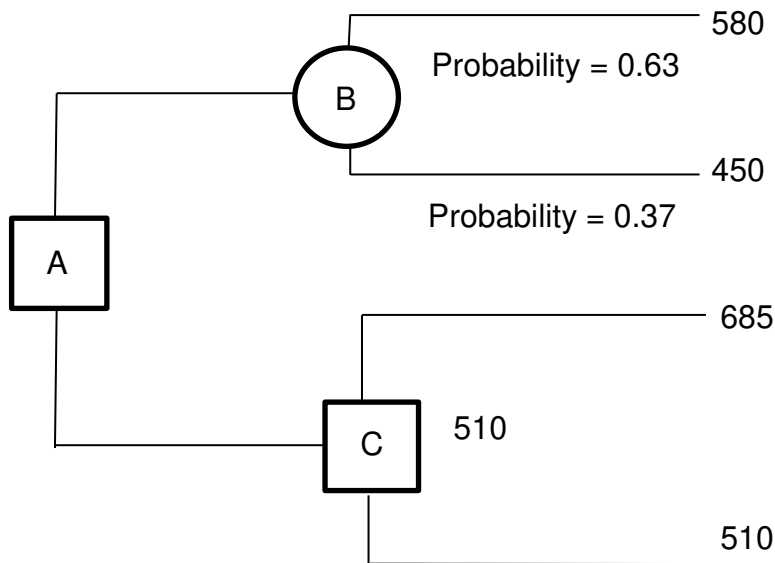
$x$	$y$	$xy$	$x^2$	$y^2$
25	89	2225	625	7921
31	71	2201	961	5041
37	79	2923	1369	6241
39	68	2652	1521	4624
42	67	2814	1764	4489
46	52	2392	2116	2704
$\Sigma x = 220$	$\Sigma y = 426$	$\Sigma xy = 15207$	$\Sigma x^2 = 8356$	$\Sigma y^2 = 31020$

$$\begin{aligned} r &= \frac{6(15207) - 220(426)}{\sqrt{(6(8356) - 220^2) \times (6(31020) - 426^2)}} \\ &= -\frac{2478}{\sqrt{8061984}} = -0.873 \text{ (to 3 sig fig.)} \end{aligned}$$

**So, the Pearson Correlation Coefficient is -0.873 (to 3 sig fig)**

- Award 1 mark for  $\Sigma x$
- Award 1 mark for  $\Sigma y$
- Award 1 mark for  $\Sigma xy$
- Award 1 mark for  $\Sigma x^2$
- Award 1 mark for  $\Sigma y^2$
- Award 1 mark for correct workings in calculation of  $r$  and 1 mark for correct value of  $r$ .

d) The following *decision tree* has been created by a management team.



- i) Roll back the decision tree **and** find the values at nodes A **and** B. In finding the value for node A you will need to decide how the *optimisation rule* is being used. 4

**Mark scheme**

**Node B is an event or chance node, so is calculated using expected value. Value at node B =  $0.63(580) + 0.37(450)$   
= 531.9**

**(1 mark for correct workings. 1 mark for numerically correct answer.)**

**Node A is a decision node and is valued by an optimum choice. Since node C has been valued at 510, the optimisation rule is being used to select the minimum value. Value at node A =  $\min [531.9, 510] = 510$**

**(1 mark for recognising that the optimisation rule is being used to select the minimum value.**

**1 mark for calculating value at Node A)**

**Total 20 Marks**

**Section B****Answer any FOUR (4) questions from this section****Marks****Question 2**

A bakery produces two flavours of cake – A and B. The available demand is known for the following week.

Cake	Demand (no of cakes)
A	42
B	29

The company must also supply a minimum of 15 type A cakes for an existing contract with a regular customer.

The production of the cakes requires two manufacturing processes: the mixing phase and the baking phase. There are no input supply constraints.

The mixing phase has 210 hours available each month, and the baking phase has 350 hours available each month. The manufacturing times and profit for each type of cake are given below:

	A	B
Mixing phase (time per cake)	15 minutes	12 minutes
Baking phase (time per cake)	50 minutes	35 minutes
Profit per cake	£3.50	£4.10

- a) The company wishes to plan for the production of cakes for the following week with the intention of **maximising** profit. Formulate the problem as an objective function and associated set of inequalities. **You are not required to obtain a numerical solution to the problem.**

**Mark scheme**

**Let A and B denote the weekly number of the two types of cake**

**Maximise:  $3.5A + 4.1B$**

**(Award 2 marks. If incorrect answer, caused by minor error award 1 mark.)**

**Subject to:**

**Cake A demand:  $A \leq 42$**

**Cake B demand:  $B \leq 29$**

**Contract for cake A:  $A \geq 15$**

**(Award 1 mark for one simple constraint, award 2 marks for all three simple constraints.)**

**Mixing phase:  $\frac{15A}{60} + \frac{12B}{60} \leq 210$  (may be simplified)**

**(1 mark partially correct, 2 marks correct inequality)**

**Baking phase:  $\frac{50A}{60} + \frac{35B}{60} \leq 350$  (may be simplified)**

**(1 mark partially correct, 2 marks correct inequality)**

**Non-negativity:  $B \geq 0$**

**(Award 1 mark for inequality. If incorrect due to minor error award 0.5 marks.)**

- b) The bakery decides to produce two more types of cake, C and D.

In order to plan for production next week with the intention of maximising profits, they formulate the problem as an objective function. They then use the Excel solver routine to solve the problem.

- i) According to the 'Answer' report, the time available for baking is a *binding constraint*. Explain what this means.

2

**Mark scheme**

**The optimal solution is constrained by the time available for baking (1 mark). If the time available for baking was changed, the optimal solution would change (1 mark).**

**Marks**  
**2**

- ii) According to the 'Sensitivity' report, the shadow price of the mixing time is zero. Explain why this is the case.

**Mark scheme**

**The shadow price of the mixing time is zero because at the optimal solution, not all of the time has been used up (1 mark).**

**If it had been beneficial to use more time for mixing the optimal solution would have done so (1 mark).**

- iii) The 'Sensitivity' report shows that the baking time for the cakes has a shadow price of £5.60. Explain what this means. **2**

**Mark scheme**

**This means that the total profit would increase by £5.60 if one more unit of baking time was made available (1 mark) and the amount of all other resources remained the same (1 mark).**

- c) The bakery uses two ovens to bake the cakes, oven A and oven B.

Oven A bakes 45% of the cakes and oven B bakes 55% of the cakes.

The probability that oven A undercooks a cake is 0.02

The probability that oven B undercooks a cake is 0.03

- i) A cake is chosen at random. Calculate the probability that it will be undercooked. **2**

**Mark scheme**

**First, define the notation for the event.**

**Let A be the event that the cake was baked in oven A.**

**Let B be the event that the cake was baked in oven B.**

**Let U be the event that the cake was undercooked.**

**Using the theorem of total probability:**

$$\begin{aligned} P(U) &= P(U|A)P(A) + P(U|B)P(B) \\ &= (0.02 \times 0.45) + (0.03 \times 0.55) \\ &= 0.0255 \end{aligned}$$

**1 mark for correct workings for calculation of P(U) and 1 mark for correct value of P(U).**

- ii) Given that a cake selected at random is undercooked, find the probability that the cake was baked in oven B (showing your workings).

**Mark scheme**

**From the theorem of Bayes:**

$$\begin{aligned}
 P(B|U) &= \frac{P(U|B)P(B)}{P(U)} \\
 &= \frac{0.03 \times 0.55}{0.0255} \\
 &= 0.647 \text{ (to 3 sf)}
 \end{aligned}$$

**Award 1 mark for partially correct method and 2 marks for wholly correct method for calculation of  $P(B|U)$  and 1 mark for correct value of  $P(B|U)$ .**

**Total 20 Marks**

**Question 3**

- a) A factory, which produces rice, records the amount of rice in 10 bags as follows:

500g 498g 502g 507g 483g 495g 491g 493g 496g 501g

- i) Calculate the *range* of the amount of rice in these 10 bags. **1**

**Mark scheme**

**Range of amount of rice:  $507g - 483g = 24g$  (1 mark)**

- ii) Calculate the *median* amount of rice in these 10 bags. **1**

**Mark scheme**

**Median amount of rice:  $\frac{496+498}{2}g = 497g$  (1 mark)**

- iii) Calculate the *mean* amount of rice in these 10 bags. Show your workings. **1**

**Mark scheme**

**The mean is:**

$$\begin{aligned}
 \bar{x} &= \frac{500 + 498 + 502 + 507 + 483 + 495 + 491 + 493 + 496 + 501}{10} \\
 &= 496.6 \text{ g}
 \end{aligned}$$

**(1 mark)**



- iv) Calculate the *sample variance* and *sample standard deviation* of the amount of rice in these 10 bags.

**Mark scheme**

**Sample variance:**  $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
500	3.4	11.56
498	1.4	1.96
502	5.4	29.16
507	10.4	108.16
483	-13.6	184.96
495	-1.6	2.56
491	-5.6	31.36
493	-3.6	12.96
496	-0.6	0.36
501	4.4	19.36
<b>TOTAL</b>		<b>402.4</b>

**Award 1 mark for mean differences.  
1 mark for mean square differences.  
1 mark for sums.**

$$s^2 = 402.4/9$$

$$= 44.7 \text{ (to 3 sig fig.)}$$

**1 mark for correct variance.**

**Standard deviation:**

$$s = +\sqrt{402.4/9} =$$

$$= 6.69 \text{ (to 3 sig fig)}$$

**Award 1 mark for correct standard deviation.**

- b) The mean amount of rice in a bag is supposed to be 500g, but the factory suspects that a machine is underfilling the bags. Determine whether there is evidence to support this at the 5% significance level. You need to state the null **and** alternative hypothesis, the critical value of the test statistic **and** your conclusions. You should assume a normal distribution. Show your workings.

**Mark scheme**

**Null hypothesis, H0:**

**The mean average amount of rice in the bags is 500g**

**H0:  $\mu = 500$**

**(1 mark)**

**Alternative hypothesis, H1:**

**The mean average amount of rice in the bags is less than 500g**

**H1:  $\mu < 500$**

**(1 mark)**

**The critical value of the test statistic is**

**$t_{0.05} (n - 1) = t_{0.05} (9) = -1.83$  (to 3 sig fig)**

**That is, if  $T < -1.83$  then H0 is rejected in favour of H1.**

**(Award 2 marks for correct numerical answer.**

**Award 1 mark for correct working but error in either calculating degrees of freedom or reading value from table.)**

**The test statistic is:**

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{496.6 - 500}{6.69/\sqrt{10}}$$

$$= -1.61 \text{ (to 3 sig fig)}$$

**(2 marks.)**

**(Award 2 marks for numerically correct answer, 1 mark for numerically incorrect answers due to minor errors)**

**The test statistic is greater than the critical value,  $-1.61 > -1.83$ . At the 5% significance level there is insufficient evidence to reject the null hypothesis. So, at the 5% significance level, there is not enough evidence to support the alternative hypothesis that the mean average amount of rice in the bags is less than 500g. (2 marks)**

**(If student has made errors in calculating the test statistic but rejects/ accepts either H0 or H1 correctly based on that statistic award full 2 marks here. Only penalise students once for errors in the calculation where the actual error is made.)**

- c) Explain what is meant by a *Type 1 error* **and** how the chance of this type of error can be minimised. 2

**Mark scheme**

**A Type 1 error occurs when the null hypothesis is incorrectly rejected.**  
(1 mark)

**The chance of a Type 1 error occurring can be minimised by lowering the significance level ( $\alpha$ ).**  
(1 mark)

- d) Explain what is meant by a *Type 2 error* **and** how the chance of this type of error can be minimised. 2

**Mark scheme**

**A Type 2 error occurs when the null hypothesis is incorrectly accepted.**  
(1 mark).

**The chance of a Type 2 error occurring can be minimised by increasing sample size. (1 mark)**

**Total 20 Marks**

**Question 4**

- a) A bookshop has recorded their book sales volume in hundreds (00s) over a FOUR (4) year period. The data is shown in the table below. 20
- i) Complete the analysis below using an *additive decomposition model* and *CMA 4*. With the aid of sketch graphs, comment upon the seasonality and trend.

Year	Quarter	Y Sales volume (00s)	T (CMA 4)
2017	Q1	78	
	Q2	69	
	Q3	75	76.625
	Q4	83	77.625
2018	Q1	81	78.75
	Q2	74	79.625
	Q3	79	80.375
	Q4	86	81.375
2019	Q1	84	82.5
	Q2	79	83.625
	Q3	83	
	Q4	91	
2020	Q1	89	
	Q2	82	
	Q3	85	
	Q4	96	

**Mark scheme**

Year	Quarter	Y Sales volume (00s)	T (CMA 4)	Y-T	S	S + T
2017	Q1	78				
	Q2	69				
	Q3	75	76.625	-1.625	-1.625	75
	Q4	83	77.625	5.375	5.042	82.667
2018	Q1	81	78.75	2.25	2.083	80.833
	Q2	74	79.625	-5.625	-5.208	74.417
	Q3	79	80.375	-1.375	-1.625	78.75
	Q4	86	81.375	4.625	5.042	86.417
2019	Q1	84	82.5	1.5	2.083	84.583
	Q2	79	83.625	-4.625	-5.208	78.417
	Q3	83	84.875	-1.875	-1.625	83.25
	Q4	91	85.875	5.125	5.042	90.917
2020	Q1	89	86.5	2.5	2.083	88.583
	Q2	82	87.375	-5.375	-5.208	82.167
	Q3	85				
	Q4	96				

**Continue CMA 4 calculations for final 4 values.**

**Possible workings:**

$$\begin{aligned}
 T(2019\ Q3) &= \frac{1}{8} (84 + 2(79) + 2(83) + 2(91) + 89) \\
 &= 84.875
 \end{aligned}$$

$$\begin{aligned}
 T(2019\ Q4) &= \frac{1}{8} (79 + 2(83) + 2(91) + 2(89) + 82) \\
 &= 85.875
 \end{aligned}$$

$$\begin{aligned}
 T(2020\ Q1) &= \frac{1}{8} (83 + 2(91) + 2(89) + 2(82) + 85) \\
 &= 86.5
 \end{aligned}$$

$$\begin{aligned}
 T(2020\ Q2) &= \frac{1}{8} (91 + 2(89) + 2(82) + 2(85) + 96) \\
 &= 87.375
 \end{aligned}$$

**(Award 1 mark for each correct CMA 4 calculation. Total of 4 marks available)**

**Calculate column of Y – T values, for example:**

$$75 - 76.625 = -1.625$$

**(For Y - T calculations marks award up to 3 marks. For errors deduct 1 mark up to a maximum of 3 marks. If error due to incorrect value of T or S but calculation correct do not deduct marks here.)**

**Possible workings for calculation of columns of S values.**

$$s_3 = \frac{(-1.625)+(-1.375)+(-1.875)}{3} = -1.625$$

$$s_4 = \frac{5.375+4.625+5.125}{3} = 5.042 \text{ (to 3 dp)}$$

$$s_1 = \frac{2.25+1.5+2.5}{3} = 2.083 \text{ (to 3 dp)}$$

$$s_2 = \frac{(-5.625)+(-4.625)+(-5.375)}{3} = -5.208 \text{ (to 3 dp)}$$

**For S calculations award 1 mark per correct calculation up to a maximum of 4 marks. If error due to incorrect values of Y - T but calculation correct do not deduct marks here.**

**Award an additional 2 marks for all values entered into correct places in table. Deduct 1 mark per error up to a maximum of 2 marks.**

**Calculate columns of S + T values, for example:**

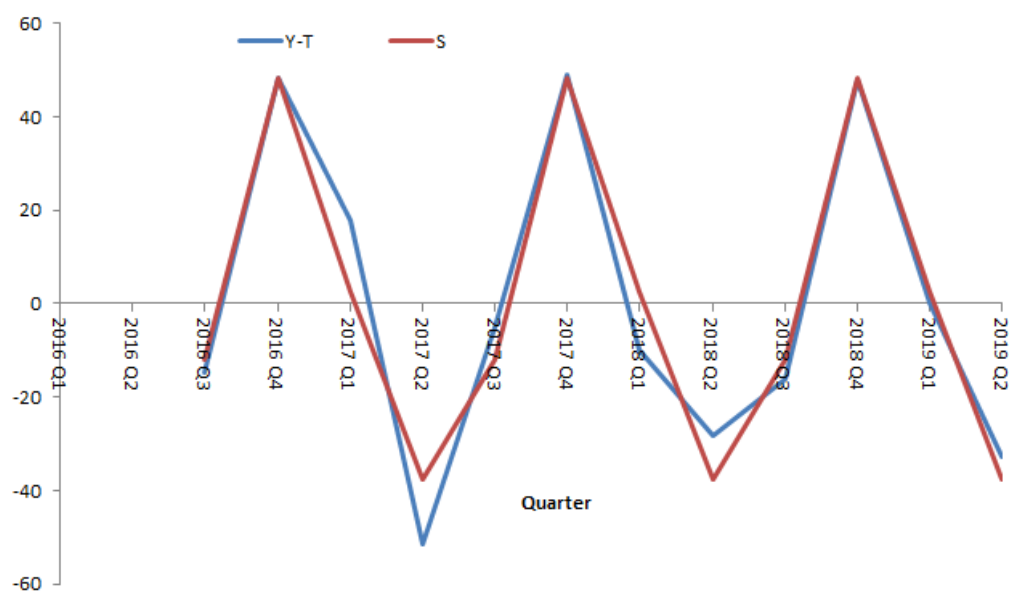
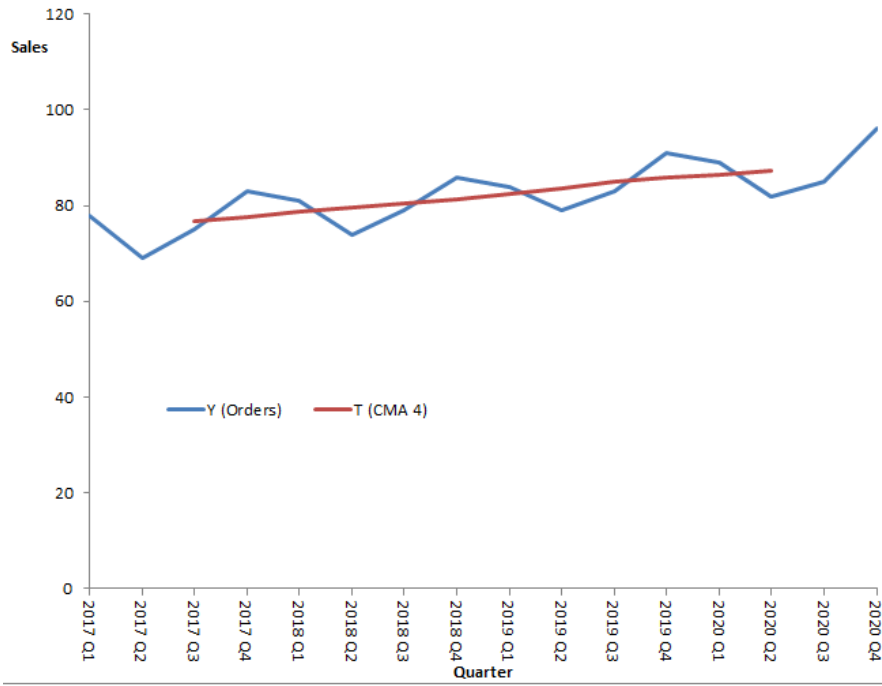
$$76.625 + (-1.625) = 75$$

**N.B. Values in S +T column of table have been rounded to 3 decimal places where necessary.**

**(For S + T calculations award up to 3 marks. For errors deduct 1 mark up to a maximum of 3 marks. If error due to incorrect value of T or S but calculation correct do not deduct marks here.)**

**Award up to 2 marks for valid observations. For example:  
Analysis appears to show an upwards trend in orders (1 mark)  
There appears to be a high season in Q4 and a low season in Q2 (1 mark)**

**Award up to 2 marks for a sketch of a graph. Sketches of possible graphs are shown below. The graphs are sketches so do not need to be accurate, however they do need to show main features. Award 1 mark for each curve on the graph.**



Total 20 Marks

## Question 5

- a) A company has developed a model for its demand curve:

$$P(q) = 58900 - 310q$$

Where  $P(q)$  denotes the unit price in GBP (£) and  $q$  the quantity of items manufactured **and** sold.

- i) Find an expression for *total revenue*,  $R(q)$ . Show your workings. 2

**Mark scheme**

**Total revenue  $R(q)$  is given by:**

$$\begin{aligned} R(q) &= q \times P(q) \\ &= 58\,900q - 310q^2 \end{aligned}$$

**1 mark for correct method, 1 mark for correct answer.**

- ii) Differentiate the expression for the total revenue,  $R(q)$ , to find the gradient of  $R(q)$ . Show your workings. 2

**Mark scheme**

$$R'(q) = 58\,900 - 620q$$

**1 mark for each correctly differentiated element.**

- iii) Find the coordinates of the turning point of  $R(q)$ . Show your workings. 4

**Mark scheme**

**The turning point occurs where  $R'(q) = 0$   
i.e. where  $58\,900 - 620q = 0$  (1 mark)  
Hence  $q = 95$  (1 mark)**

**When  $q = 95$**   

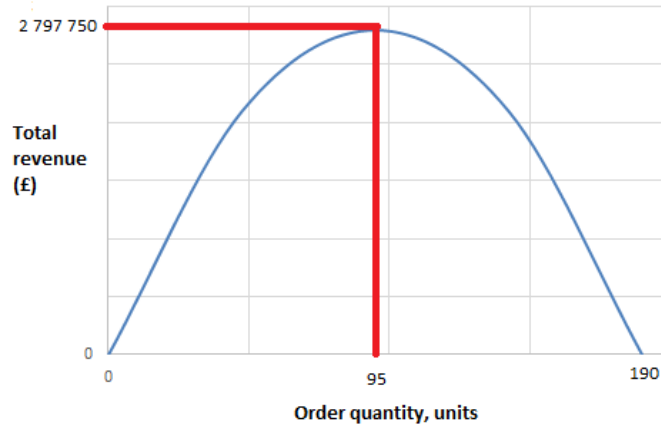
$$R(q) = (58\,900 \times 95) - 310(95)^2 = \pounds 2\,797\,750 \quad (1 \text{ mark})$$

**Hence the turning point occurs at (95, 2 797 750) (1 mark)**

- iv) Sketch a graph of total revenue against output. You should label the axes **and** the turning point. State the maximum total revenue.

**Mark scheme**

*Coordinates of turning point (95, 2 797 750)*



**Award 1 mark for a graph showing a maximum turning point and 1 mark for correctly labelled axes.**

**Award 1 mark for correctly labelled turning point.**

**Hence, the maximum revenue is £2 797 750. (1 mark)**

- b) 12 learner drivers are sitting their driving test. The probability of passing the driving test is 0.7. Calculate the probability that **at least** 10 of the learner drivers will pass the test. 4

**Mark scheme**

**Using the binomial distribution (1 mark for using binomial distribution)**

$$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

$$n = 12$$

$$p = 0.7$$

$$P(X = 10) + P(X = 11) + P(X = 12)$$

$$= \frac{12!}{10!(2)!} (0.7)^{10} (0.3)^2 + \frac{12!}{11!(1)!} (0.7)^{11} (0.3)^1 + \frac{12!}{12!(0)!} (0.7)^{12} (0.3)^0$$

**(1 mark for partially correct method, 2 marks for wholly correct method)**  
**= 0.253 (to 3 sig fig)**

**(1 mark for correct answer)**



- c) A clothing manufacturer forecasts next season's demand for a coat as: expected demand 1450 and a standard deviation 300. Assume demand is normally distributed. Calculate the probability that the actual demand will be between 1330 and 1585. **4**

**Mark scheme**

**Calculate z – score,  $z = (x - \mu) / \sigma$**

$$= \frac{1330-1450}{300}$$

$$= -0.4$$

**(Award 1 mark for numerically correct score)**

**Calculate z – score,  $z = (x - \mu) / \sigma$**

$$= \frac{1585-1450}{300}$$

$$= 0.45$$

**(Award 1 mark for numerically correct score.)**

**Use tables  $P(-0.4 < z < 0.45) = 0.6736 - 0.3446$**

$$= 0.329$$

**(Award 1 mark for correct method and 1 mark for correct answer. If numerically incorrect answer due to errors in calculating z scores award 2 marks as FT.)**

**Total 20 Marks**

## Question 6

- a) Six candidates are interviewed for a job. The two interviewers rank the candidates as follows:

Candidate	A	B	C	D	E	F
Interviewer 1	5	1	2	4	6	3
Interviewer 2	6	3	1	2	5	4

- i) Calculate the *Spearman Correlation Coefficient*. Show your workings.

5

**Mark scheme****Using**

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

<b>Candidate</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>
<b>Interviewer 1</b>	<b>5</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>3</b>
<b>Interviewer 2</b>	<b>6</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>5</b>	<b>4</b>
<b><i>d</i></b>	<b>-1</b>	<b>-2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>-1</b>
<b><i>d</i><sup>2</sup></b>	<b>1</b>	<b>4</b>	<b>1</b>	<b>4</b>	<b>1</b>	<b>1</b>

$$r_s = 1 - \frac{6 \times 12}{6 \times 35} = 1 - \frac{72}{210} = 0.657 \text{ (to 3 sig fig.)}$$

**1 mark for *d* values**

**1 mark for *d*<sup>2</sup> values**

**1 mark for sum**

**1 mark for use of formula**

**1 mark for correct answer**

- ii) Comment on the value of the *Spearman Correlation Coefficient*.

2

**Mark scheme**

**The Spearman Rank Correlation Coefficient indicates quite a strong positive relationship between the ranks assigned to the candidates by the two interviewers.**

**(Award 1 mark for positive and 1 mark for 'quite strong' or similar)**

- b) A survey is carried out in a large city. 686 people out of a random sample of 980 said that they had shopped online in the past month. Determine a 99% confidence interval for the proportion of people in the city that have shopped online in the past month.

**Mark scheme**

**Proportion =  $686/980 = 0.7$  (1 mark)**

**$np$  and  $n(1 - p)$  are both greater than 5 so large sample CI for proportion can be used (1 mark).**

**From tables  $z_{99\%} = 2.5758$  (1 mark)**

$$\begin{aligned}
 (p^-, p^+) &= \left( \bar{p} - Z_{\gamma} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + Z_{\gamma} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \right) \\
 &= \left( 0.7 - 2.5758 \sqrt{\frac{0.7 \times 0.3}{980}}, 0.7 + 2.5758 \sqrt{\frac{0.7 \times 0.3}{980}} \right) \\
 &= (0.662, 0.738) \text{ (to 3 sig fig)}
 \end{aligned}$$

**Award 1 mark for workings.**

**Award 1 mark for correct value.**

**(If value incorrect due to incorrect calculation of either  $p$  or  $z_{99\%}$  award 2 marks as FT)**

**With 99% confidence, between 66.2% and 73.8% of the people in the city have shopped online in the past month.**

**(1 mark. If percentages incorrect due to incorrect calculations of  $(p^-, p^+)$  but interpretation correct award 1 mark.)**

- c) A price index initially used 2015 as its base year. In 2019 the base year was updated.

YEAR	PRICE INDEX (2015 BASE)	PRICE INDEX (2019 BASE)
2015	100	
2016	101.8	
2017	102.7	
2018	104.9	
2019	106.3	100
2020		101.2

- i) Link the two series to create a single series with 2019 as the base year. Give your answers to ONE (1) decimal place.

**5**

**Mark scheme**

YEAR	PRICE INDEX (2015 BASE)	PRICE INDEX (2019 BASE)
2015	100	<b>94.1</b>
2016	101.8	<b>95.8</b>
2017	102.7	<b>96.6</b>
2018	104.9	<b>98.7</b>
2019	106.3	100
2020		101.2

**Possible workings:**

**2015:**  $100 \times \frac{100}{106.3} = 94.1$  (to 1 d.p)

**2016:**  $101.8 \times \frac{100}{106.3} = 95.8$  (to 1 d.p)

**2017:**  $102.7 \times \frac{100}{106.3} = 96.6$  (to 1 d.p)

**2018:**  $104.9 \times \frac{100}{106.3} = 98.7$  (to 1 d.p)

**Award 1 mark for correct method.**

**Award 1 mark for each numerically correct answer.**

**Maximum of 5 marks.**

- ii) Use the single series to calculate the percentage change from 2017 to 2020. Give your answer to ONE (1) decimal place.

**2**

**Mark scheme**

**Percentage change from 2017 to 2020** =  $100 \times \frac{101.2 - 96.6}{96.6} = 4.8\%$  (to 1 d.p.)  
**(Award 1 mark for correct working. Award 1 mark for numerically answer.)**

**Total 20 Marks**

**End of paper**

<b>Formula sheet</b>
<b>Management statistics</b>

Population mean and standard deviation

$$\mu = \frac{\sum f_i x_i}{N} \qquad \mu = \sum p_i x_i$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{N}} \qquad \sigma = \sqrt{\sum p_i (x_i - \mu)^2}$$

Population Coefficient of Variation

$$CV = \frac{\sigma}{\mu}$$

Sample mean, standard deviation and sample variance

$$\bar{x} = \frac{\sum f_i x_i}{n} \qquad s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n - 1}} \qquad s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Sample skewness

$$\frac{n}{(n - 1)(n - 2)} \sum_{i=1}^n \left( \frac{x_i - \bar{x}}{s} \right)^3$$

Sample Coefficient of Variation

$$CV = \frac{s}{\bar{x}}$$

Simple Index Number

$$R = \frac{p_n}{p_0} \times 100$$

Laspeyres and Paasche Price Index Numbers

$$LPI = 100 \times \frac{\sum q_o p_n}{\sum q_o p_o}$$

$$PPI = 100 \times \frac{\sum q_n p_n}{\sum q_n p_o}$$

Laspeyres and Paasche Quantity Index Numbers

$$LQI = 100 \times \frac{\sum q_n p_o}{\sum q_o p_o}$$

$$PQI = 100 \times \frac{\sum q_n p_n}{\sum q_o p_n}$$

## Probability

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Theorem of Bayes

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)}$$

Theorem of Total Probability

$$P(A) = \sum_i P(A|E_i)P(E_i)$$

Binomial Distribution  $B(n, p)$

$$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

$$\mu = np \quad \sigma = \sqrt{np(1-p)}$$

Poisson Distribution  $Po(\lambda)$

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\mu = \lambda \quad \sigma = \sqrt{\lambda}$$

Exponential Distribution

$$f(t) = \lambda e^{-\lambda t}, \quad t \geq 0$$

$$P(T < t) = 1 - e^{-\lambda t}$$

$$\mu = \frac{1}{\lambda} \quad \sigma = \frac{1}{\lambda}$$

Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

$$f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

## Hypothesis Testing

Distribution of sample means

$$\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Large sample confidence interval of the mean ( $n \geq 30$ )

$$(\mu^-, \mu^+) = \left( \bar{x} - z_{\gamma} \frac{s}{\sqrt{n}}, \bar{x} + z_{\gamma} \frac{s}{\sqrt{n}} \right)$$

Large sample confidence interval of proportion ( $np$  and  $n(1 - p) \geq 5$ )

$$(p^-, p^+) = \left( \bar{p} - z_{\gamma} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}, \bar{p} + z_{\gamma} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \right)$$

Exact confidence interval (underlying population has normal distribution)

$$(\mu^-, \mu^+) = \left( \bar{x} - t_{\gamma} \frac{s}{\sqrt{n}}, \bar{x} + t_{\gamma} \frac{s}{\sqrt{n}} \right)$$

Approximate large sample test of the mean

$$Z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Under the null hypothesis  $Z \sim N(0, 1)$ , approximately.

Student's one sample t-test of the mean.

$$T = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Under the null hypothesis  $T \sim t(n - 1)$

Independent two sample t-test

$$T = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Under the null hypothesis  $T \sim t(n_1 + n_2 - 2)$

## Fitting Data

$\chi^2$  Goodness of fit test ( $E_i \geq 5$  for all  $i$ )

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

Under the null hypothesis  $\chi^2 \sim \chi^2(k - m - 1)$

$k$  is number of categories,  $m$  is number of model parameters estimated from data

$\chi^2$  Test of Association ( $E_i \geq 5$  for all  $i$ )

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

Under the null hypothesis  $\chi^2 \sim \chi^2((r - 1)(c - 1))$

$r$  is number of rows,  $c$  is number of columns

Simple Linear Regression

$$\hat{y} = mx_i + c$$

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$c = \bar{y} - m\bar{x}$$

$$m = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

is the least SSE straight line where;

The Coefficient of Determination

$$R^2 = r^2 = \frac{\sum(\hat{y} - \bar{y})^2}{\sum(y - \bar{y})^2}$$

The Pearson Correlation Function

$$R = r = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{(n \sum x_i^2 - (\sum x_i)^2)(n \sum y_i^2 - (\sum y_i)^2)}}$$

Spearman's Rank Correlation (with no ties)

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$



## Differentiation

### Definition

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

### Standard Derivatives

$y$	$\frac{dy}{dx}$
$y = ax^n$	$\frac{dy}{dx} = nax^{n-1}$
$y = e^{ax}$	$\frac{dy}{dx} = ae^{ax}$
$y = \ln(ax)$ $= \log_e(x)$	$\frac{dy}{dx} = \frac{1}{x}$

### Rules of Differentiation

$$\frac{d}{dx} (af(x) + bg(x)) = a \frac{df}{dx} + b \frac{dg}{dx}$$

$$\frac{d}{dx} (f(x)g(x)) = f(x) \frac{dg}{dx} + g(x) \frac{df}{dx}$$

$$\frac{d}{dx} (f(g(x))) = \frac{df}{dg} \frac{dg}{dx}$$

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{df}{dx} - f(x) \frac{dg}{dx}}{(g(x))^2}$$

### Elasticities of Demand

Own price	Cross price	Income
$E_p = \frac{\partial Q_1}{\partial p_1} \frac{p_1}{Q_1}$	$E_{12} = \frac{\partial Q_1}{\partial p_2} \frac{p_2}{Q_1}$	$E_I = \frac{\partial Q_1}{\partial I} \frac{I}{Q_1}$

### The Total Differential

$$y = y(x_1 + x_2 + x_3 + \dots)$$

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \frac{\partial y}{\partial x_3} dx_3 + \dots$$

$$\Delta y \approx \frac{\partial y}{\partial x_1} \Delta x_1 + \frac{\partial y}{\partial x_2} \Delta x_2 + \frac{\partial y}{\partial x_3} \Delta x_3 + \dots$$

## Time series

The additive decomposition model

$$Y_n = T_n + S_n + I_n$$

The multiplicative decomposition model

$$Y_n = T_n \times S_n \times I_n$$

Three Point Moving Average

$$T_n = \frac{1}{3}(Y_{n-1} + Y_n + Y_{n+1})$$

Four Point Centred Moving Average

$$T_n = \frac{1}{8}(Y_{n-2} + 2Y_{n-1} + 2Y_n + 2Y_{n+1} + Y_{n+2})$$

Simple Exponential Smoothing

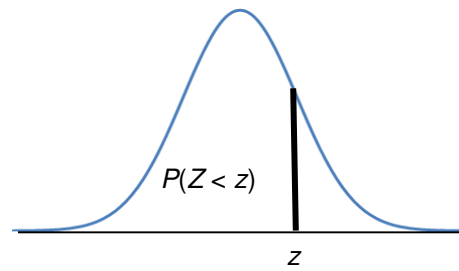
$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

Errors

$$MSE = \frac{1}{N} \sum_{j=1}^N (Y_j - F_j)^2$$

$$MAE = \frac{1}{N} \sum_{j=1}^N |Y_j - F_j|$$

## Probabilities under the Normal Distribution Curve

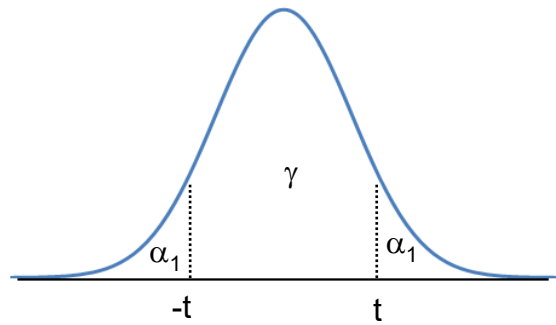


<b>z</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>-3.50</b>	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
<b>-3.40</b>	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
<b>-3.30</b>	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
<b>-3.20</b>	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
<b>-3.10</b>	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
<b>-3.00</b>	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
<b>-2.90</b>	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
<b>-2.80</b>	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
<b>-2.70</b>	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
<b>-2.60</b>	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
<b>-2.50</b>	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
<b>-2.40</b>	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
<b>-2.30</b>	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
<b>-2.20</b>	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
<b>-2.10</b>	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
<b>-2.00</b>	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
<b>-1.90</b>	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
<b>-1.80</b>	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
<b>-1.70</b>	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
<b>-1.60</b>	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
<b>-1.50</b>	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
<b>-1.40</b>	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
<b>-1.30</b>	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
<b>-1.20</b>	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
<b>-1.10</b>	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
<b>-1.00</b>	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
<b>-0.90</b>	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
<b>-0.80</b>	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
<b>-0.70</b>	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
<b>-0.60</b>	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
<b>-0.50</b>	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
<b>-0.40</b>	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
<b>-0.30</b>	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
<b>-0.20</b>	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
<b>-0.10</b>	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
<b>-0.00</b>	<b>0.5000</b>	<b>0.4960</b>	<b>0.4920</b>	<b>0.4880</b>	<b>0.4840</b>	<b>0.4801</b>	<b>0.4761</b>	<b>0.4721</b>	<b>0.4681</b>	<b>0.4641</b>

<b>z</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0.00</b>	<b>0.5000</b>	<b>0.5040</b>	<b>0.5080</b>	<b>0.5120</b>	<b>0.5160</b>	<b>0.5199</b>	<b>0.5239</b>	<b>0.5279</b>	<b>0.5319</b>	<b>0.5359</b>
<b>0.10</b>	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
<b>0.20</b>	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
<b>0.30</b>	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
<b>0.40</b>	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
<b>0.50</b>	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
<b>0.60</b>	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
<b>0.70</b>	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
<b>0.80</b>	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
<b>0.90</b>	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
<b>1.00</b>	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
<b>1.10</b>	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
<b>1.20</b>	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
<b>1.30</b>	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
<b>1.40</b>	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
<b>1.50</b>	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
<b>1.60</b>	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
<b>1.70</b>	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
<b>1.80</b>	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
<b>1.90</b>	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
<b>2.00</b>	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
<b>2.10</b>	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
<b>2.20</b>	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
<b>2.30</b>	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
<b>2.40</b>	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
<b>2.50</b>	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
<b>2.60</b>	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
<b>2.70</b>	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
<b>2.80</b>	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
<b>2.90</b>	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
<b>3.00</b>	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
<b>3.10</b>	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
<b>3.20</b>	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
<b>3.30</b>	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
<b>3.40</b>	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
<b>3.50</b>	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

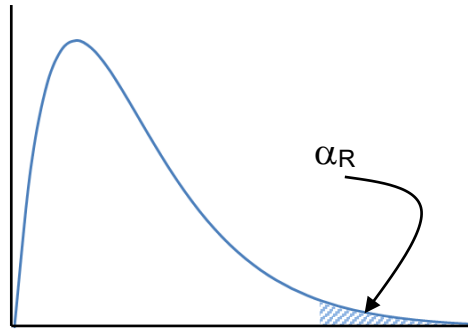
## Percentage Points of the Student t Distribution

$\alpha_1$	5.00%	2.50%	1.00%	0.50%	
$\alpha_2$	10.00%	5.00%	2.00%	1.00%	
$\gamma$	90.00%	95.00%	98.00%	99.00%	
<b>df</b>	1	6.3138	12.7062	31.8205	63.6567
	2	2.9200	4.3027	6.9646	9.9248
	3	2.3534	3.1824	4.5407	5.8409
	4	2.1318	2.7764	3.7469	4.6041
	5	2.0150	2.5706	3.3649	4.0321
	6	1.9432	2.4469	3.1427	3.7074
	7	1.8946	2.3646	2.9980	3.4995
	8	1.8595	2.3060	2.8965	3.3554
	9	1.8331	2.2622	2.8214	3.2498
	10	1.8125	2.2281	2.7638	3.1693
	11	1.7959	2.2010	2.7181	3.1058
	12	1.7823	2.1788	2.6810	3.0545
	13	1.7709	2.1604	2.6503	3.0123
	14	1.7613	2.1448	2.6245	2.9768
	15	1.7531	2.1314	2.6025	2.9467
	16	1.7459	2.1199	2.5835	2.9208
	17	1.7396	2.1098	2.5669	2.8982
	18	1.7341	2.1009	2.5524	2.8784
	19	1.7291	2.0930	2.5395	2.8609
	20	1.7247	2.0860	2.5280	2.8453
	21	1.7207	2.0796	2.5176	2.8314
	22	1.7171	2.0739	2.5083	2.8188
	23	1.7139	2.0687	2.4999	2.8073
	24	1.7109	2.0639	2.4922	2.7969
	25	1.7081	2.0595	2.4851	2.7874
	26	1.7056	2.0555	2.4786	2.7787
	27	1.7033	2.0518	2.4727	2.7707
	28	1.7011	2.0484	2.4671	2.7633
	29	1.6991	2.0452	2.4620	2.7564
	30	1.6973	2.0423	2.4573	2.7500
	31	1.6955	2.0395	2.4528	2.7440
	32	1.6939	2.0369	2.4487	2.7385
	33	1.6924	2.0345	2.4448	2.7333
	34	1.6909	2.0322	2.4411	2.7284
	35	1.6896	2.0301	2.4377	2.7238
	36	1.6883	2.0281	2.4345	2.7195
	37	1.6871	2.0262	2.4314	2.7154
	38	1.6860	2.0244	2.4286	2.7116
	39	1.6849	2.0227	2.4258	2.7079
	40	1.6839	2.0211	2.4233	2.7045
	$\infty$	<b>1.6449</b>	<b>1.9600</b>	<b>2.3263</b>	<b>2.5758</b>



## Critical Values for the $\chi^2$ Distribution

df	$\alpha_R$	0.05	0.01
		5.00%	1.00%
1		3.841	6.635
2		5.991	9.210
3		7.815	11.345
4		9.488	13.277
5		11.070	15.086
6		12.592	16.812
7		14.067	18.475
8		15.507	20.090
9		16.919	21.666
10		18.307	23.209
11		19.675	24.725
12		21.026	26.217
13		22.362	27.688
14		23.685	29.141
15		24.996	30.578
16		26.296	32.000
17		27.587	33.409
18		28.869	34.805
19		30.144	36.191
20		31.410	37.566
21		32.671	38.932
22		33.924	40.289
23		35.172	41.638
24		36.415	42.980
25		37.652	44.314
26		38.885	45.642
27		40.113	46.963
28		41.337	48.278
29		42.557	49.588
30		43.773	50.892



## Learning Outcomes matrix

Question	Learning Outcomes assessed	Marker can differentiate between varying levels of achievement
1	1,2,3,4	Yes
2	1	Yes
3	2, 4	Yes
4	3	Yes
5	1,2,4	Yes
6	1,2,3	Yes

## Grade descriptors

Learning Outcome	Pass	Merit	Distinction
Use summary and inferential statistics to inform business decisions.	Demonstrate adequate and appropriate use of statistics.	Demonstrate appropriate and effective use of statistics.	Demonstrate highly appropriate and effective use of statistics.
Analyse management decisions using optimisation techniques.	Demonstrate adequate ability to analyse decisions.	Demonstrate ability to provide detailed and coherent analysis of decisions.	Demonstrate ability to provide comprehensive, lucid analysis of decisions.
Understand and apply approaches to business forecasting.	Demonstrate ability to perform the task.	Demonstrate ability to perform the task consistently well.	Demonstrate ability to perform the task to the highest standard.
Evaluate sequential management decisions.	Provide a reasonable assessment of the subject; Ideas are generally coherent.	Provide a generally strong assessment with some well-reasoned assumptions; Ideas are consistently coherent.	Provide a consistently strong assessment with well-reasoned and original assumptions; All ideas are highly coherent.